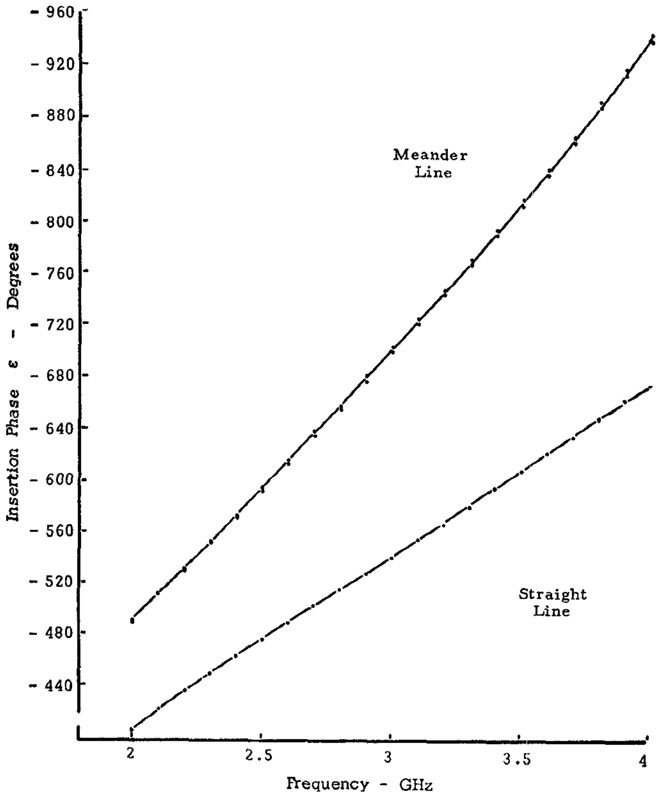

Fig. 7. Insertion phase versus frequency for a line with two meanders and a straight line both on a 2-in ferrite G500 substrate.

coefficient S_{21} is shown for the two latched directions of magnetization. Although S_{12} measurements are not shown, they were within experimental error of 1° from S_{21} values with the latching reversed. At higher frequencies the meander line shows the beginning of cutoff, for S_{11} was measured as 0.5 at 5.25 GHz. S_{11} becomes 0.7 at 5.6 GHz. There is evidence indicating that coupling one basic meander to another causes a shift in the predicted cutoff frequency.

There is no known direct way to calculate nonreciprocal differential phase shift. Thus Fig. 6 shows the measured phase shift of this experimental line at frequencies in S band. Measurements on a straight line deposited on the same substrate allow the insertion phase of the meander line alone to be determined.
These measurements are shown in Fig. 7.

Conclusions

The experimental results clearly verify that a desired image impedance of a line with few meanders can be achieved using the outlined procedure. Tolerance in dimensions shows that tradeoff is possible between strip spacing and leg width for the same meander line Z_I. The theoretical bandpass characteristic predicted is supported by the tendency for the line to show cutoff at 5.25 GHz. With only two tightly coupled meanders operating near 3 GHz, the data show 10° of switchable nonreciprocal differential phase shift to be possible. The precise effect on cutoff frequency and impedance of coupling one basic meander to another together with the demonstration of a way to calculate meander-line nonreciprocal differential phase shift remain subjects for further research.

Acknowledgment

The author wishes to thank C. Blake of Lincoln Laboratory for making the research possible, and Dr. J. A. Weiss, Consultant to Lincoln Laboratory, for his valuable advice and criticism.

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Mode Chart for Microstrip Ring Resonators

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***Abstract*—A universal mode chart for the microstrip ring resonator, based on a radial waveguide model, is presented. The resonant frequency is related to the width of the ring conductor. Experimental results from 4 to 16 GHz are shown to be in good agreement with the theory.**

Introduction

Ring resonators have found application in circulators, hybrid junctions, filters, and other microstrip devices. Troughton [1] and Caulton *et al* [2]. have used them to measure propagation constants in microstrip, based on the principle that the ring is resonant when its mean circumference equals an integral number of wavelengths. Wolff and Knoppik [3] have recently shown that there are dispersive effects that must be considered, as well as the influence of the ring center conductor width on the resonant frequency. This short paper presents a universal mode chart for ring resonators, relating the resonant frequency to the ring width and its mean circumference. Qualitative agreement is shown with experimental results. Quantitative agreement may be obtained by modifying the effective dielectric constant of the substrate according to Wheeler's approximation [4].

Theory

Fig. 1 shows the geometry of the resonator whose mean radius is $R=(a+b)/2$. If the electromagnetic fields are assumed to be confined to the dielectric volume between the perfectly conducting ground plane and the ring conductor, then the fields may be shown to be TM to z [3]. The field components are E_z, H_r, and H_ϕ and the resonant modes are denoted as TM_{nml}. For the thin substrates normally encountered in microwave integrated circuits, the fields may be taken as independent of the z coordinate ($l=0$).

The fields propagate in the radial direction and may have ϕ variation. Application of the usual magnetic wall boundary conditions at $r=a$; $r=b$, typically assumed for wide microstrip [5], [6] leads to the well-known characteristic equation for the resonant modes:

$$J_n'(ka)Y_n'(kb) - J_n'(kb)Y_n'(ka) = 0 \qquad (1)$$

with

$$k = \frac{\omega}{c}\sqrt{\epsilon_d} \qquad (2)$$

where ω is the resonant radian frequency, c is the speed of light in vacuo, and ϵ_d is the relative dielectric constant of the substrate. The quantities $J_n(x)$ and $Y_n(x)$ are Bessel's functions of the first and second kind, order n, respectively, and the prime denotes derivatives with respect to kr. The integer n is the azimuthal mode number. Notice that (1) is the same equation satisfied by TE modes in coaxial waveguides [7]. The use of idealized boundary conditions is discussed later.

For narrow microstrip widths ($a \simeq b$), (1) reduces to

$$[(ka)^2 - n^2][J_{n-1}(ka)Y_n(ka) - Y_{n-1}(ka)J_n(ka)] = 0. \qquad (3)$$

Manuscript received May 12, 1972; revised February 26, 1973. This work was sponsored by the United States Air Force Avionics Laboratory, WPAFB, Dayton, Ohio, under Contract F33615-72-C-1054.
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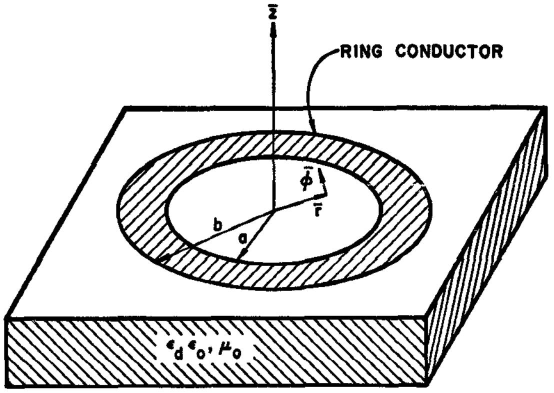

Fig. 1. Geometry of microstrip ring resonator.

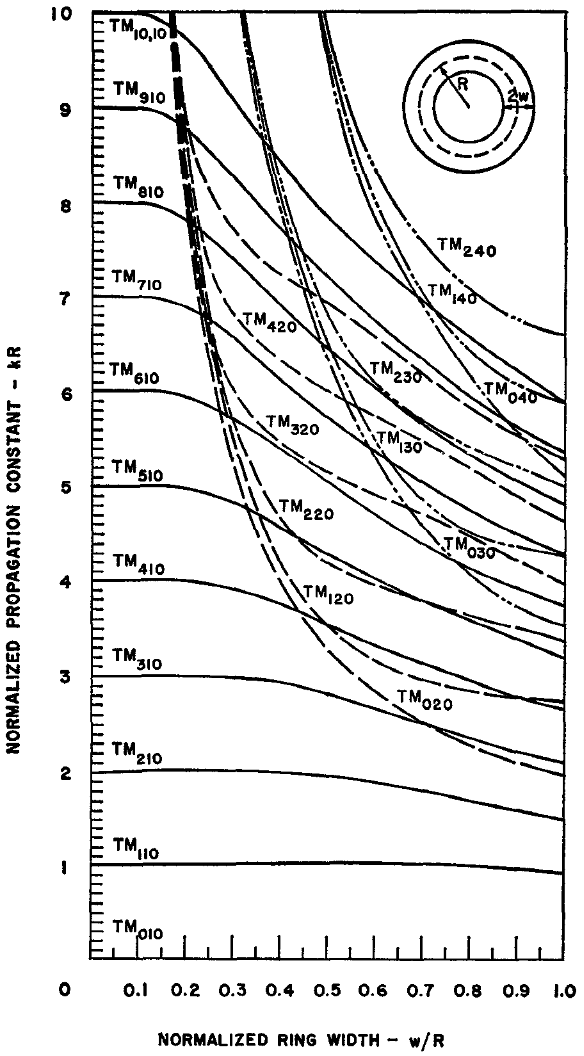

Fig. 2. Mode chart for microstrip ring resonator.

Since the second factor is nonzero, the solution for narrow ($m=1$) ring widths is the well-known result [2] $ka=n$.

Discussion

Fig. 2 is the mode chart for the TM_{nmo} resonant modes which relates the radial propagation constant k and mean radius R, to the width of the microstrip center conductor $2w=b-a$. The index m designates the mth radial root of (2) for a given n. The roots ka of (2) are found numerically as a function of kb and displayed as kR versus w/R.

Several features of Fig. 2 may be noted. For narrow width rings ($w/R\rightarrow 0$) the resonant modes are TM_{n10} modes with $kR=n$. These are the modes used in the measurement of microstrip propagation constants [1], [2]. As w/R is increased, the value of kR becomes less than n for a given mode. When the ring width reaches half the guided wavelength, higher order TM_{nmo} modes ($n\geq 0$, $m>1$) appear. Finally as the inner radius shrinks to zero, $(w/R\rightarrow 1)r$, the ring becomes a disk resonator [8]. For example, the dipolar TM_{110}-mode resonance is given by $k(2R)=1.84$.

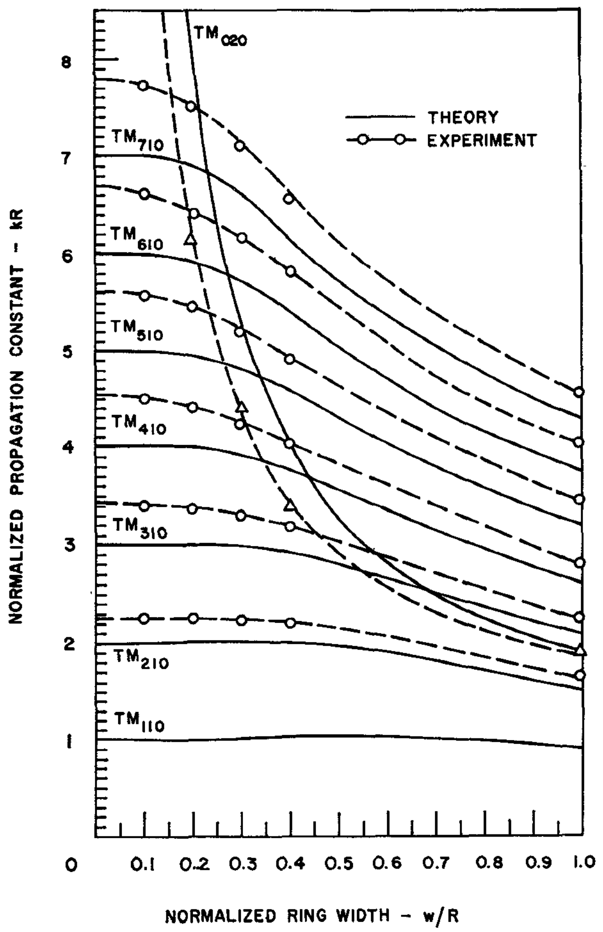

Fig. 3. Experimental results.

Experiments

In order to validate the behavior predicted in Fig. 2, microstrip ring resonators with fixed mean radius ($R=0.8$ cm), but with various widths $0.1\leq w/R\leq 0.4$; $w/R=1$, were fabricated on 0.025-in-thick alumina ($\epsilon_d=9.9$) substrates. Each resonator was excited by a lightly coupled 50-Ω microstrip line. Resonances were observed in the frequency range 4–16 GHz by monitoring transmission on a second lightly coupled 50-Ω output line colinear with the input. The experimental results are shown in Fig. 3 where ϵ_d is used in the calculation of k.

The dominant TM_{110} mode was not seen, since on the experimental rings used its resonance is near 2 GHz, below the range of available sources. Numerous higher order TM_{mno} modes were found, as expected from Fig. 2, but were not shown in Fig. 3 for the sake of clarity.

The experimental TM_{n10}-mode curves are displaced above the theoretical curves in Fig. 3. If Wheeler's effective dielectric constant [4]

$$\epsilon_{eff}=1+q(\epsilon_d-1)$$

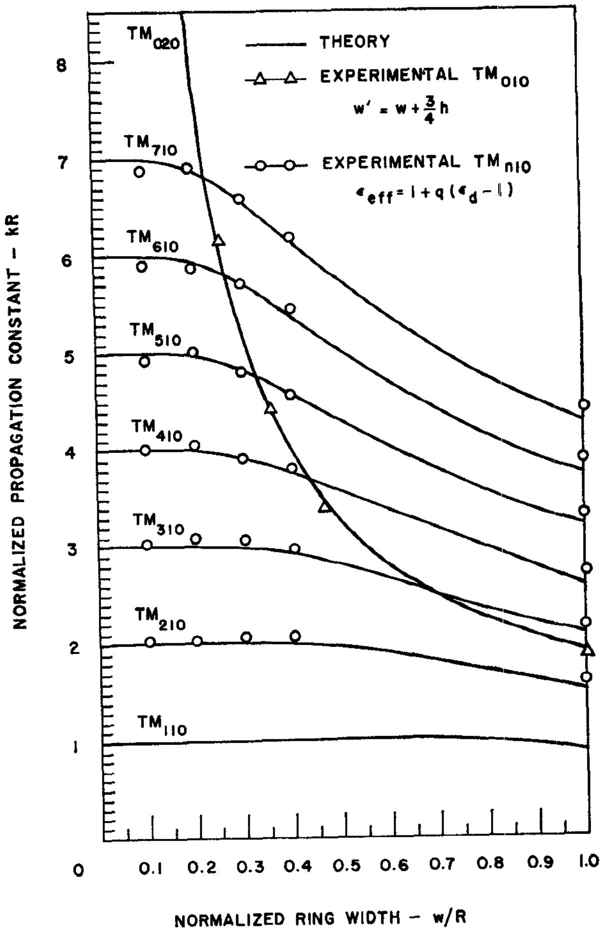

Fig. 4. Comparison of corrected experimental results with theory.

TABLE I

Corrected Dielectric Constant Ratio

Normalized Ring Width $\frac{w}{R}$	Width To Height Ratio $\frac{2w}{h}$	Filling Factor (4) q	$\sqrt{\frac{\epsilon_{eff}}{\epsilon_d}}$
0.1	2.52	.77	.89
0.2	5.04	.84	.92
0.3	7.56	.86	.93
0.4	10.08	.89	.94
1.0	25.2	.95	.97

is used to calculate k, the experimental points are brought into good agreement with the theory as shown in Fig. 4. Table I shows the value of q, and the ratio $\sqrt{\epsilon_{eff}/\epsilon_d}$ for the various ring widths used in the experiment.

Also shown in Table I is the ring width to substrate height ratio $2w/h$. Even for the narrowest ring used, this ratio exceeds 2.5. Thus it is felt that the assumption of magnetic wall boundary conditions and, in fact, the use of ϵ_{eff} is well justified. Dispersive effects, such as those reported by Wolff and Knoppik, were noted only for the narrowest ring $w/R=0.1$. As seen in Fig. 3 they are most prominent

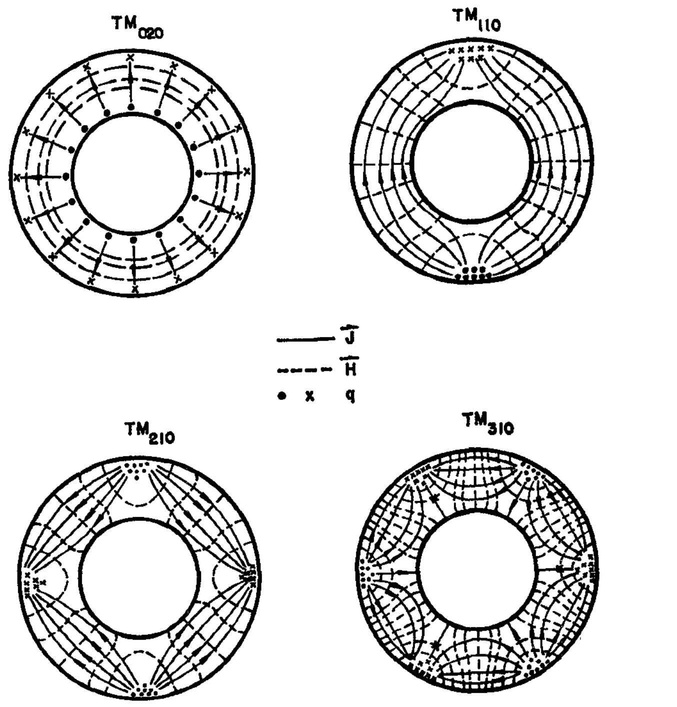

Fig. 5. Field patterns of some ring resonator modes.

for the high values of n (high frequencies), as expected. In this work, the mean radius was fixed and the inner and outer radii changed, in contrast to Wolff and Knoppik who kept the outer radius constant. The highest resonator characteristic impedance used here was approximately 35 Ω.

Field patterns for some of the lower modes are shown in Fig. 5. These modes are the duals of the TE waveguide modes in coaxial waveguide [9]. The TM_{020} mode has a significant H_ϕ component and is by no means a TEM mode from the microstrip viewpoint. In this mode, high electric fields at the ring edges produce fringing which increases the effective ring width as seen in Fig. 3. The empirical correction

$$w' = w + 3/4\, h$$

where h is the substrate height, produces the agreement shown in Fig. 4 for the TM_{020} mode.

No width corrections were applied to the disk resonator results ($w/R=1$), but ϵ_{eff} was used. The frequency difference between the corrected experimental results and theory is approximately $\Delta f/f \simeq 3$ percent, with the experimental results lying above the theoretical results. This indicates that a lower ϵ_{eff} than suggested by Wheeler is needed to fit the theoretical results.

The mode chart of Fig. 2 shows that the TM_{110} mode is the dominant mode of the ring resonator for any ring width. If wide rings are used, higher order TM_{nmo} modes can be supported. These modes may be avoided for $w/R \leq 0.1$. However, for thin rings, the effect of dispersion must be considered in the application of ring resonators [3].

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